S2 Poisson Distⁿ Past Qu.s

1) Jan 2005 no.1

A secretary is typing a document. The number of typing mistakes the secretary makes per page can be modelled by a Poisson distribution with mean 2. Find the probability that

(i) in a document consisting of one page, the secretary makes more than 3 mistakes, [2]

(ii) in a document consisting of two pages, the secretary makes a total of fewer than 6 mistakes. [2]

2) Jan 2006 no.1

In a study of urban foxes it is found that on average there are 2 foxes in every 3 acres.

(i) Use a Poisson distribution to find the probability that, at a given moment,

(a) in a randomly chosen area of 3 acres there are at least 4 foxes, [2]

(b) in a randomly chosen area of 1 acre there are exactly 2 foxes. [3]

(ii) Explain briefly why a Poisson distribution might not be a suitable model. [2]

3) Jan 2009 no.3

The number of incidents of radio interference per hour experienced by a certain listener is modelled by a random variable with distribution Po(0.42).

(i) Find the probability that the number of incidents of interference in one randomly chosen hour is

(a) 0,

(b) exactly 1.

[3]

(ii) Find the probability that the number of incidents in a randomly chosen 5-hour period is greater than 3. **[3]**

(iii) One hundred hours of listening are monitored and the numbers of 1-hour periods in which 0, 1, 2, … incidents of interference are experienced are noted. A bar chart is drawn to represent the results. Without any further calculations, sketch the shape that you would expect for the bar chart. (There is no need to use an exact numerical scale on the frequency axis.) **[2]**

4) June 2009 no.5

In a large region of derelict land, bricks are found scattered in the earth.

(i) State two conditions needed for the number of bricks per cubic metre to be modelled by a Poisson distribution. **[2]**

Assume now that the number of bricks in 1 cubic metre of earth can be modelled by the distribution Po(3).

(ii) Find the probability that the number of bricks in 4 cubic metres of earth is between 8 and 14 inclusive. **[3]**

(iii) Find the size of the largest volume of earth for which the probability that no bricks are found is at least 0.4. **[4]**

5) June 2010 no.1

(i) The number of inhabitants of a village who are selected for jury service in the course of a 10-year period is a random variable with the distribution Po(4.2).

(a) Find the probability that in the course of a 10-year period, at least 7 inhabitants are selected for jury service. **[2]**

(b) Find the probability that in 1 year, exactly 2 inhabitants are selected for jury service. **[3]**

(ii) Explain why the number of inhabitants of the village who contract influenza in 1 year can probably not be well modelled by a Poisson distribution. **[2]**

6) June 2010 no.6

(a) The random variable D has the distribution Po(24). Use a suitable approximation to find $P(D > 30)$. **[5]**

(b) An experiment consists of 200 trials. For each trial, the probability that the result is a success is 0.98, independent of all other trials. The total number of successes is denoted by E.

(i) Explain why the distribution of E cannot be well approximated by a Poisson distribution. **[1]**

(ii) By considering the number of failures, use an appropriate Poisson approximation to find $P(E \leqslant 194)$. **[4]**

7) June 2001 no.7

Over a long period it has been found that the number of typing errors made by a secretary is, on average, 6 per hour.

(i) State an assumption which you need to make in order to model the number of typing errors in a randomly chosen hour by a Poisson distribution. [1]

Assuming that this model is valid, and that the relevant periods of time are randomly chosen,

(ii) calculate the probability that, in one hour, the secretary makes more than 5 typing errors, [2]

(iii) calculate the probability that, in ten minutes, the secretary makes exactly two typing errors, [3]

(iv) use a suitable approximation to calculate the probability that, in four hours, the secretary makes more than 30 typing errors, [5]

(v) find the longest period of time, in minutes, for which the probability that the secretary makes no typing errors is greater than 0.9. [3]

8) Jan 2002 no.8

In a survey of supermarket queues, it is observed that customers arrive at the checkouts independently of one another. The average rate at which customers arrive at the checkouts between 11 a.m. and 3 p.m. is taken to be a constant 3 per minute.

(i) Find the probability that exactly ten customers arrive at the checkouts between 1.00 p.m. and 1.05 p.m. **[3]**

For the period between 11 a.m. and 3 p.m.,

(ii) use tables to estimate, correct to 2 significant figures, the longest time for which the probability that fewer than 2 customers arrive at the checkouts is greater than 0.06, **[3]**

(iii) by calculation find the range of values of t for which the probability that no customers arrive in a period of t seconds is greater than 0.000 01. **[5]**

9) June 2002 no.1

Sixty people each make two throws with a fair six-sided die. Using a suitable approximation, calculate the probability that at least four of the sixty obtain two sixes. [5]

10) Jan 2003 no.6

The number of misprints in a randomly chosen page of the Barchester *Gazette* is denoted by the random variable M. The mean value of M is 2.

(i) State two conditions needed in order to model M by a Poisson distribution. [2]

(ii) Find the probability that, in five randomly chosen pages of the *Gazette*, in the edition of 31 January 2003, there will be a total of more than 12 misprints. [3]

(iii) Find the smallest number of complete pages such that the probability that they contain a total of more than 12 misprints is greater than 0.7. You should show the values of any relevant probabilities obtained from tables. [4]

11) June 2003 no.4

Calls received by a car rescue service occur independently and at a constant average rate of 3 per minute.

(i) Find the probability that, in a randomly chosen period of 4 minutes, the number of calls received by the service is exactly 14. [3]

(ii) Find the longest period of time, in seconds to the nearest 0.1 s, for which the probability that no calls are received by the service is greater than 0.2. [4]

12) Jan 2004 no.1

The number of currants in a randomly chosen fruit scone can be modelled by a Poisson distribution with mean $4\frac{2}{3}$.

(i) Calculate the probability that, in one randomly chosen fruit scone, there are exactly 3 currants. [2]

(ii) Use Poisson tables to find the probability that, in 3 randomly chosen fruit scones, there is a total of no more than 11 currants. [2]

13) June 2004 no.6

Requests for the services of a professional services company are received at a constant average rate of 38 per week, independently of one another. If more than 50 requests are received in any one week, the company has to take on extra staff.

(i) Use a suitable approximation to show that the probability that the company has to take on extra staff in one randomly chosen week is 0.021, correct to 3 decimal places. [5]

(ii) Use a suitable approximation to find the probability that, in a year of 50 working weeks, the company has to take on extra staff in more than 2 weeks. [5]